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Exam I, MTH 418, Spring 2016

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QUESTION 1. (i) Let H be a connected graph of order n and of size $n-1$. Show (prove) that H is a tree. **Solution: Since H is connected, we know that H has a spanning subgraph that is a tree, say T. Since the order of T is n, we know that the size of T is n-1. Thus $H = T$.**

(ii) Let H be a connected graph of order n. Show that H must have a spanning subgraph that is a connected bipartite. **Solution: Since H is connected, we know that H has a spanning subgraph that is a tree, say T. Since every tree is connected d and bipartite graph, we are done.**

(iii) Let H be a connected graph of order n and of size n. Assume that H has no bridges. Show that $H = C_n$ and hence every vertex of H is of degree 2. **Solution: Since H is connected, we know that H has a spanning subgraph that is a tree, say T. Thus T is of order n and with size $n-1$. Since the size of H is n, there exists exactly one edge say e of H that is not an edge of T. Hence $H = T + e$. Note that adding exactly one edge to a tree will create one and only one cycle, say C_k, where e must be a part of C_k. We claim that $k = n$. Assume $k \neq n$. Then there is an edge in T that is not an edge in C_k (because $H = T + e$ and e is a part of C_k). Every edge in T is a bridge and thus H has a bridge, a contradiction. Thus $k = n$, and hence $H = C_n$.**

(iv) Assume that H is a disconnected graph of order n with exactly two components (say, H_1 and H_2) and with associated non-increasing sequence $d_1 \geq d_2 \geq d_3 \geq \cdots \geq d_n$. Assume that H_1 is of order $m \geq 2$ and H_2 has has a cycle. Show that there is a connected graph D of order n (the same order as H) and with associated non-increasing sequence as H. **Solution: Let e be an edge of a cycle in H_2. Hence we know by a result that e is not a bridge and thus $H_2 - e$ is CONNECTED!!!, let f be an edge in H_1. Now apply the 2-switch sequence on the edges e, f. Then we get a connected graph D. Since D is obtained from H by using a sequence of 2-switches, we KNOW that H and D must have the same non-increasing sequence....**

(v) Construct a graph of order 5 and with associated non-increasing sequence $2 \geq 1 \geq 1 \geq 1 \geq 1$? Can we construct a connected graph of order 5 and with associated non-increasing sequence $2 \geq 1 \geq 1 \geq 1 \geq 1$? Does that contradict question (iv)?explain **Solution: trivial**

(vi) Let D be a graph of order n and with associated non-increasing sequence $d_1 = 3 \geq 1 \geq 1 \geq \cdots \geq 1 = d_n$. Show that n must be even. **Solution: We know that number of vertices with odd degrees in any graph (see class notes) must be an even number. Since every vertex in D is either of degree 3 or 1, we conclude that n is even by the result.**

(vii) Let D be a graph of order 6 and with associated non-increasing sequence $d_1 = 5 \geq 3 \geq d_3 \geq d_4 \geq 1 \geq 1 = d_6$. Find all possible values of d_3 and d_4. **Use the Algorithm we studied in the class (on whether a sequence of degrees is GRAPHICAL or not), we conclude that $d_3 = d_4 = 3$ or $d_3 = d_4 = 2$**

(viii) Let H be a graph with vertex-set $= \{v_1, ..., v_{11}\}$ and D is a graph with vertex-set $= \{w_1, w_2, w_3, w_4, w_5, w_6\}$, Let $F = H \times D$ (Graph Product). Hence $(v_1, w_2), (v_3, w_4) \in V(F)$. Assume $v_1 - v_2 - v_3$ is the shortest path (walk) in H from v_1 to v_3 and $w_1 - w_2 - w_3 - w_4$ is the shortest path (walk) in D from w_1 to w_4 in D. Find the distance between (v_1, w_2) and (v_3, w_4). Construct a shortest path from $(v1, w_2)$ to (v_3, w_4).[NOTE: You do not need to construct $H \times D$] **Solution: ALL of you got it right**

(ix) We know that every tree is addressable (i.e., every tree is isomorphic to an induced subgraph of Q_k (k-cubes graph). Find the address of each vertex of the following tree of order 5. **See CLASS NOTES on how to label each vertex, note that T here is isomorphic to INDUCED subgraph of Q_4 and never isomorphic to induced subgraph of Q_3.**

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